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SRB THERMAL CURTAIN DESIGN SUPPORT

DECEMBER 1991 FINAL REPORT TO

U.S. POLYMERIC
B.P. CHEMICALS
SANTA ANA, CALIFORNIA 92705

PURCHASE ORDER NUMBER 62550

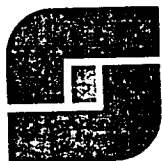
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Advanced Materials Division
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Santa Ana, California 92705

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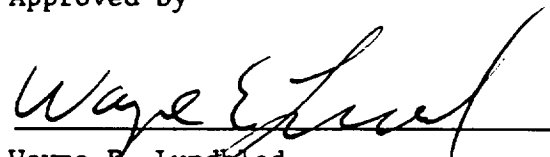
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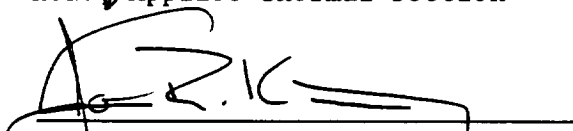


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INTRODUCTION

This is the final report for work performed for B.P. Chemicals under purchase order number 62550, NASA prime contract NAS8-38261, "SRB Thermal Curtain Redesign Support". This report covers the period from 1 April 1990 to 31 December 1991.

The objective of the program during this time period was to evaluate candidate materials that could be used to design an improved Solid Rocket Motor (SRM) Aft Skirt Thermal Curtain (ASTC). The ASTC is a flexible, high temperature, cloth and insulation composite that is used to protect the hardware located inside the aft skirt of the shuttle solid rocket booster (Figure 1). The current ASTC consists of nine layers of insulating materials and is 2.58 inches thick (Figure 2). The ASTC is made up of twenty four segments. The segments are hand sewn together during installation on the aft skirt (Figure 3). The weight of the current ASTC is approximated at about six hundred pounds. This weight does not include the weight of the mounting hardware and ties required to install the twenty four ASTC segments (which is significant).

The current effort entailed measuring the thermophysical properties of six candidate materials and then using these properties in a computer program to predict the thermal performance of various curtain configurations subjected to an SRM heat flux. The candidate materials under consideration were supplied by B.P. Chemicals and consisted of quartz, S-glass and Kevlar woven into nominal 0.25 inch thick layers by a unique process known as angle-interlock and polar-weave. The polar-weave material is a modification of the angle-interlock weave that has the advantage of being able to be woven about a radius. This type of weave would vastly simplify the construction of the ASTC by allowing for either a one-piece design or a smaller number of segments. Also, the presence of Kevlar would increase the strength of the ASTC making it less likely to tear from overpressure or flutter. Table 1 lists the physical characteristics of the six candidate materials supplied by B.P. Chemicals for thermophysical property measurements.

THERMOPHYSICAL PROPERTY MEASUREMENTS

Values of the specific heat for quartz, S-glass and Kevlar were obtained from literature¹. Figures 4 through 6 are plots of specific heat versus temperature for the aforementioned materials. Tables 2 through 4 are tabulated recommended values of specific heat.

Low temperature thermal conductivity measurements (530°R to about 1200°R) were made utilizing the ATSM C-177 guarded hot plate technique. Measurements were performed on all six materials. Figures 7 through 9 are the thermal conductivity versus temperature curves for the candidate materials. As can be seen, the thermal conductivity of the polar-weave fabric is slightly less than that of the angle-interlock fabric. This is fortunate, since the polar-weave fabric is desired for ASTC design. Tables 5 through 7 are tabulated recommended values of thermal conductivity. These tables also contain some extrapolated values that are necessary for the thermal analysis. These extrapolated values are explained later.

Optical properties (transmittance, reflectance and emittance) were measured from 1.6 to 26 microns at 530°R on the polar-weave quartz only. Transmittance data showed the polar-weave quartz to be opaque so no optical properties were required on the S-glass and Kevlar. Reflectance and emittance measurements were subcontracted to Surface Optics Corporation, San Diego, California. Reflectance measurements were taken at three incident angles in both the circumferential and radial orientations. Measurements were performed using a Cary-Integrating Sphere Reflectometer. Values of emittance were calculated from the transmittance and reflectance data. Table 8 summarizes the optical properties measurements.

COMPUTER PROGRAM DEVELOPMENT

A one-dimensional transient heat transfer computer program was developed to assist in the thermal analysis of the ASTC redesign.

An implicit, forward-differencing technique² was used with the nodal spacing taken at the interfaces of a multiple layer curtain configuration. Figure 10 shows a four node configuration for a proposed three piece curtain.

The implicit technique involves evaluating an energy balance at a node (the heat flow into a node minus the heat flow out of a node is equal to the amount of heat stored in the node). Table 9 depicts the nodal equations for a simple four node system. Since the internal nodes are actually composed of two materials the energy storage term is adjusted to reflect an average value. Table 10 is the matrix form of the nodal equations. Table 11 lists the matrix coefficients. Gauss-Jordan upper triangularization³ is used to solve the matrix. Radiant heat flux is varied as a function of time. The heat flux curve used (Figure 11)⁴ was obtained from NASA SRM thermal design data. Radiation heat transfer at node 1 is approximated by using the emittance at the surface to reduce the net heat flux accordingly.

Although Tables 9 through 11 reflect only a four node system the computer program will expand the matrix according to the number of layers selected for the configuration.

The current computer program is called "ASTC version 3.1". The program is very user friendly. All user inputs are described in detail on the prompt screens. A copy of the program on a 5-1/4" diskette is attached to the end cover of this report. An IBM compatible, 386 PC with color VGA is recommended to run this program. To use the program create a directory called "ASTC" on the PC hard drive. Change to the ASTC directory. Copy all programs from the diskette to the ASTC directory. To run the program type 'astc3'. Follow the instructions on the screen.

ANALOG TEST FACILITY

A quartz lamp thermal flux facility was developed to perform actual tests on curtain configurations constructed by B.P. Chemicals. Figure 12 is a schematic of this test facility. A sample curtain 12"x12" can be instrumented with thermocouples on the surface and between layers and then exposed to the SRM flux shown in Figure 11. A calibrated heat flux transducer can be used to control the flux from the quartz lamps. Data is recorded continuously by a data acquisition system. Data are presented in the form of temperature-time plots.

RESULTS AND CONCLUSIONS

Since the polar-weave fabric was the fabric of choice for construction of an improved ASTC, and since the thermal conductivity of the polar-weave fabric was less than or comparable to the angle-interlock fabric, it was decided to perform all further tests and analysis on curtains employing the polar-weave constructions.

Samples of polar-weave quartz, S-glass and Kevlar, cut 12"x12", were sewn together to form a three-piece curtain for testing in the quartz lamp facility. Although this curtain design is twice as heavy as the current ASTC design (about 1000 lbs versus 600 lbs, not considering hardware installation tradeoffs) the analog data from this particular design can be used to validate the computer program. The sample was instrumented with three thermocouples and exposed to the SRM design flux for 120 seconds. Three tests were run. Figure 13 shows the results for the average of the three tests.

Next the computer program was used to perform a thermal analysis on the three-piece blanket. Since elevated temperature thermal conductivity was required on the quartz and S-glass an approximation had to be made for these values (high temperature measurements are part of a future effort). Figures 14 and 15 represent the approximations selected for the most probable high temperature thermal conductivity of the quartz and S-glass. It is these values that are shown in Tables 5 and 6.

It was estimated that the incident radiation on the ASTC from the SRM plume would be approximately 45 degrees⁵. Therefore, an emittance value of 0.85 was selected (see Table 8) for the analysis.

Figure 16 presents the results of the thermal analysis for the three-piece curtain. Figure 17 is a composite drawing showing the analog test data and the thermal analysis data. As can be seen there is good agreement with the surface temperature. This leads credibility to the computer program. The analog interface temperatures are lower than the predicted temperatures probably due to the interfacial resistances present in the real test. The computer program has no provision for inputting interfacial resistances. However, the surface temperature

is the controlling design parameter. If an ASTC curtain is designed to the program predictions then the actual curtain will perform with a margin of safety. Also, the results shown in Figure 17 indicate that this type of curtain, although heavier than the current ASTC, is within the thermal performance requirements for an ASTC.

It appears that the polar-weave materials can be used to design an ASTC that will meet the thermal and strength requirements for SRM design. However, optimum cost effective design will have to take into account the installation and mounting hardware requirements.

FUTURE EFFORTS

Since the computer program appears to yield valid results it is possible to use the program to optimize proposed designs for candidate SRM thermal curtains. Efforts are currently under to review SRM/ASTC installation requirements and use the thermal analysis to evaluate some curtain configurations.

Also, efforts are underway to modify the computer program to utilize the advanced solid rocket motor (ASRM) flux and make recommendations for an ASRM/ASTC.

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4. NASA/MSFC, "SRB Thermal Design Databook" Rev. D., September 1987.
5. United Technologies/USBI, "AFT Skirt Thermal Curtain", June 1987. Report to NASA/MSFC, Contract NAS8-36300.

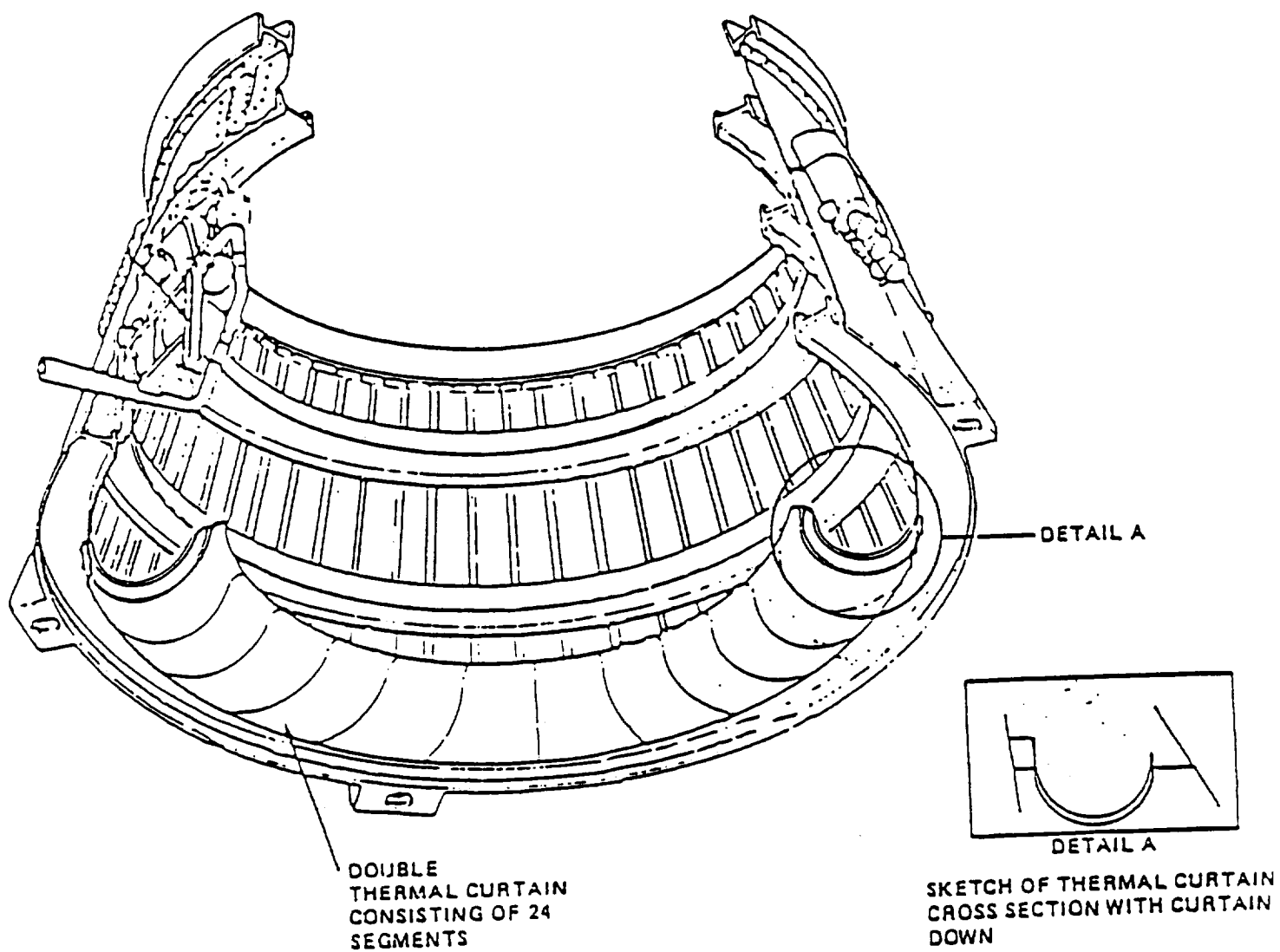


Figure 1. Location of Aft Skirt Thermal Curtain

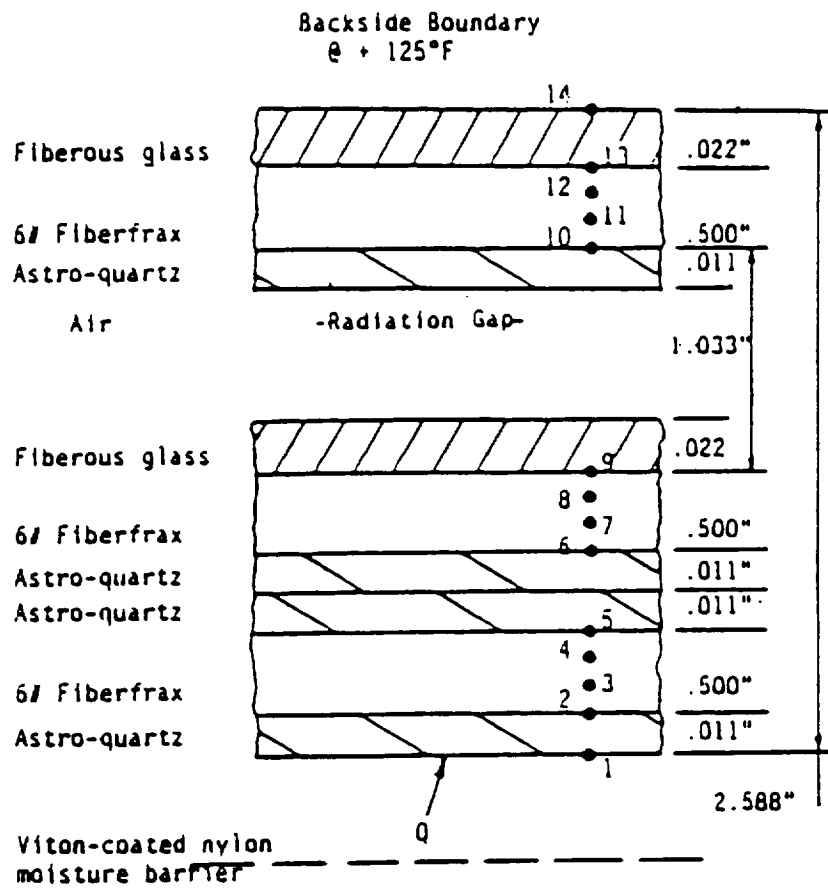
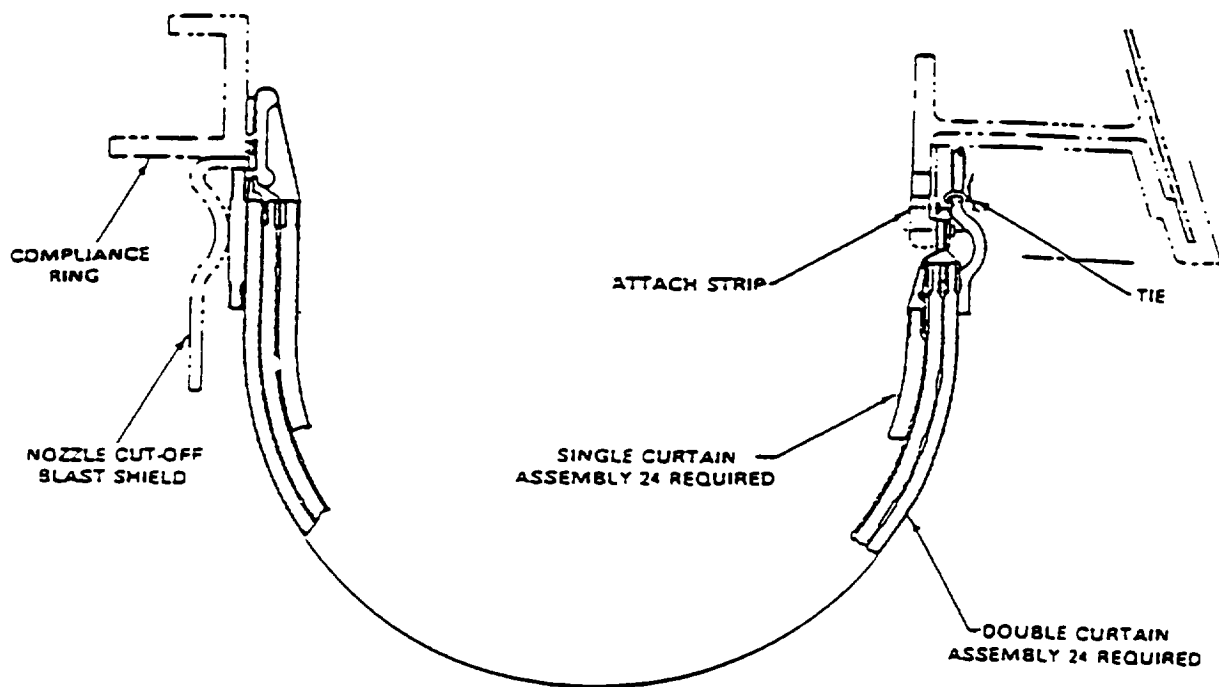


Figure 2. Aft Skirt Thermal Curtain Construction



THERMAL CURTAIN INSTALLATION

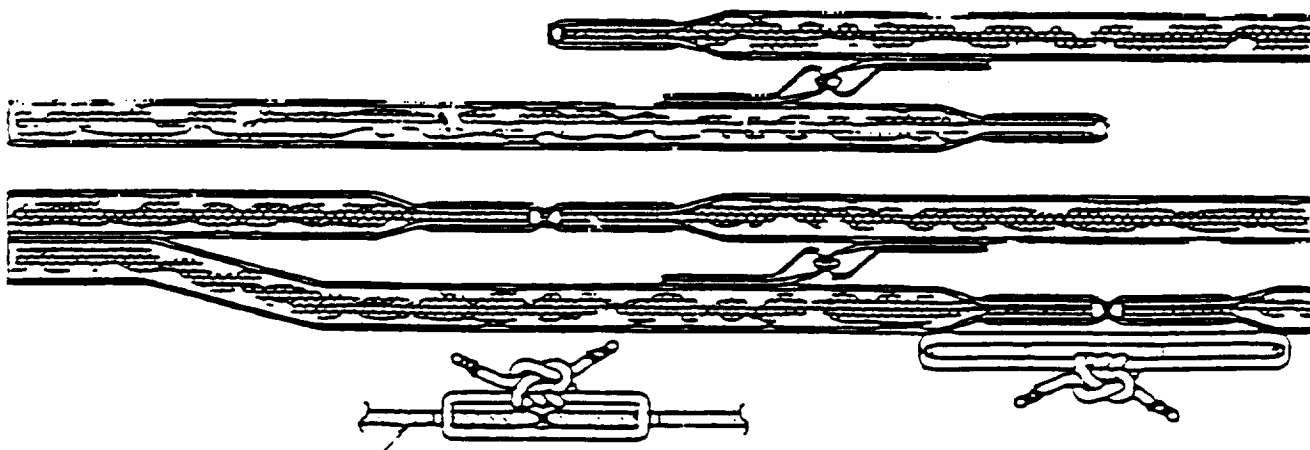


Figure 3. Aft Skirt Thermal Curtain Installation

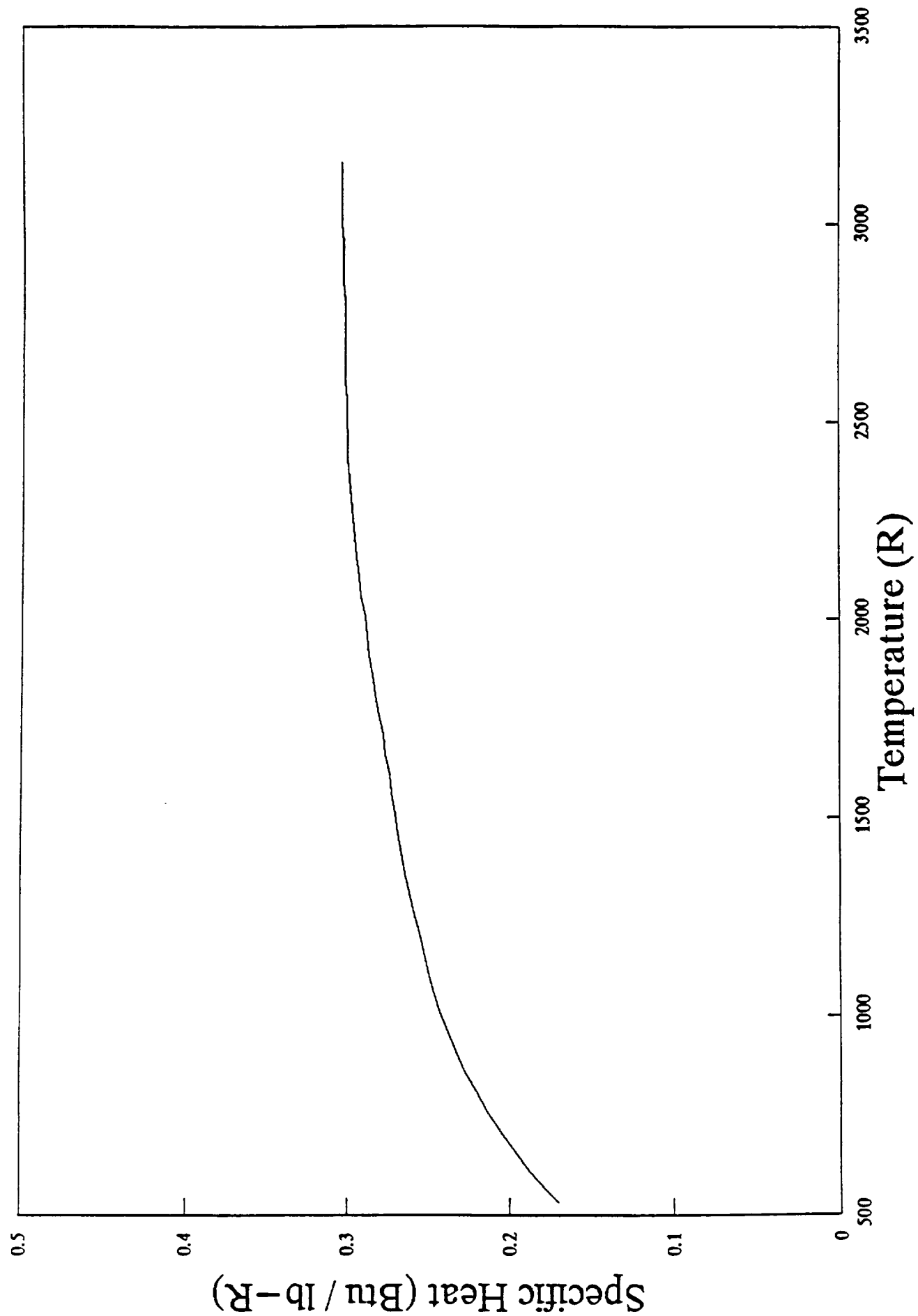


Figure 4. Specific Heat of Quartz Fabric

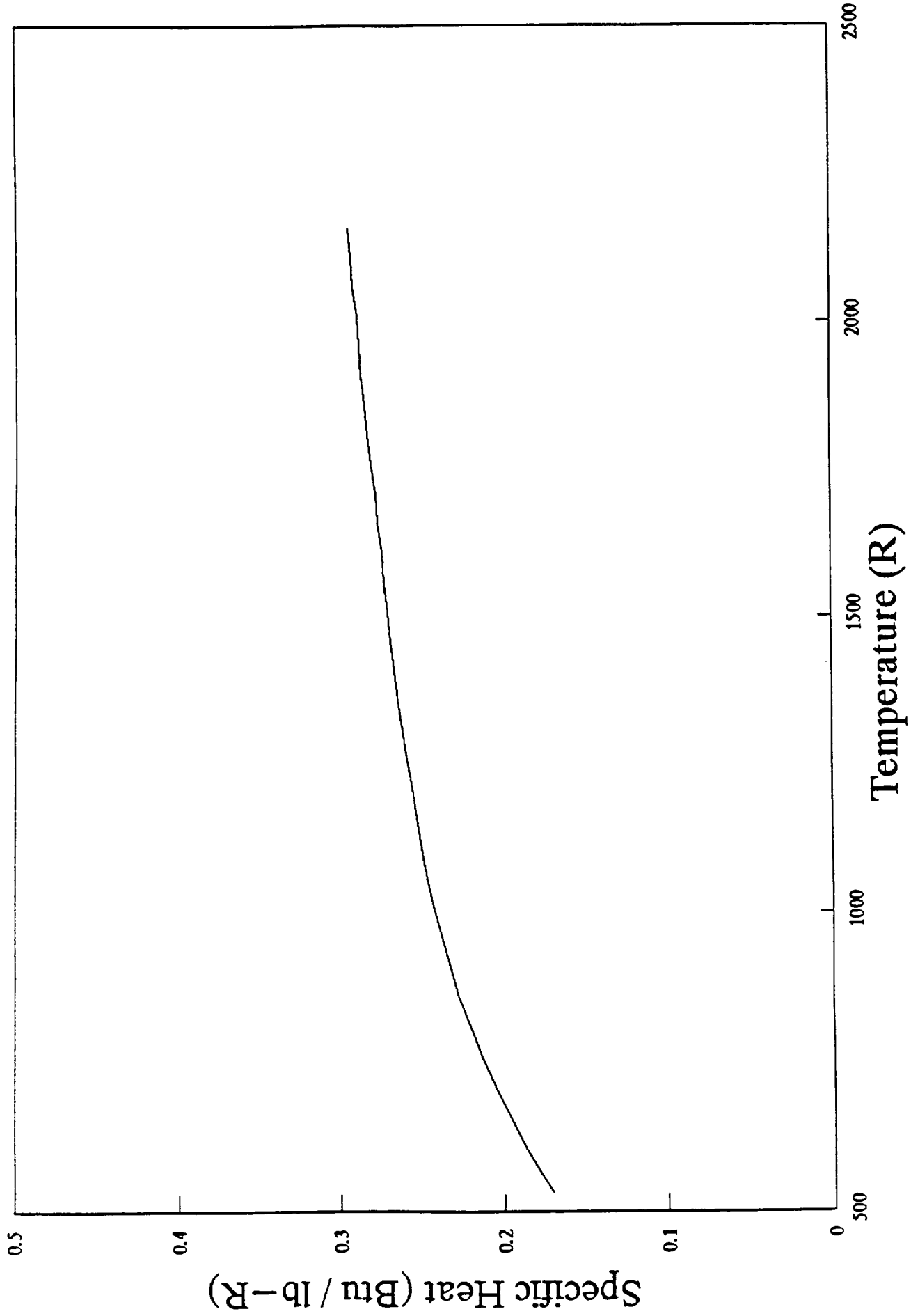


Figure 5. Specific Heat of S-Glass Fabric

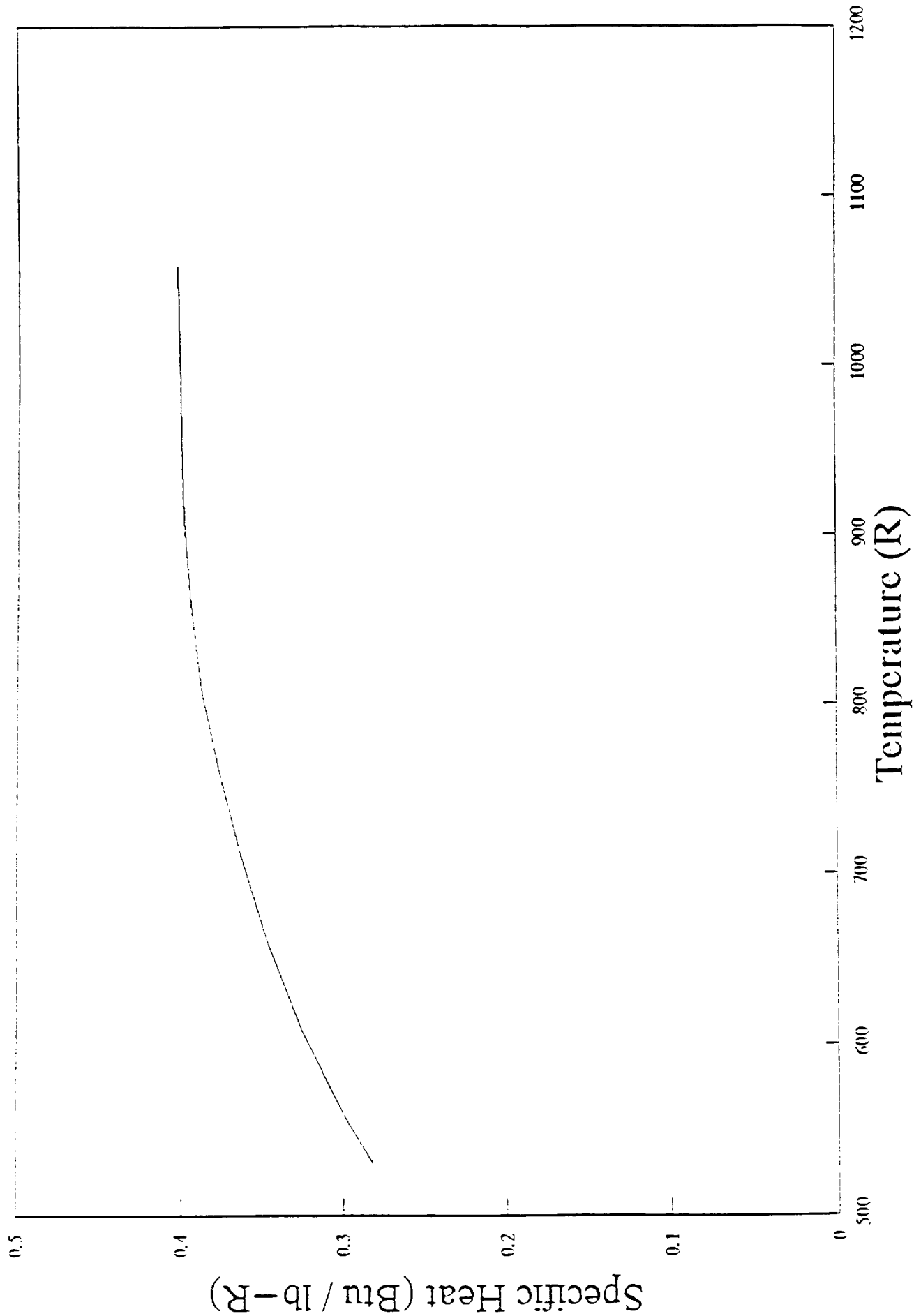


Figure 6. Specific Heat of Kevlar Fabric

Quartz Thermal Conductivity

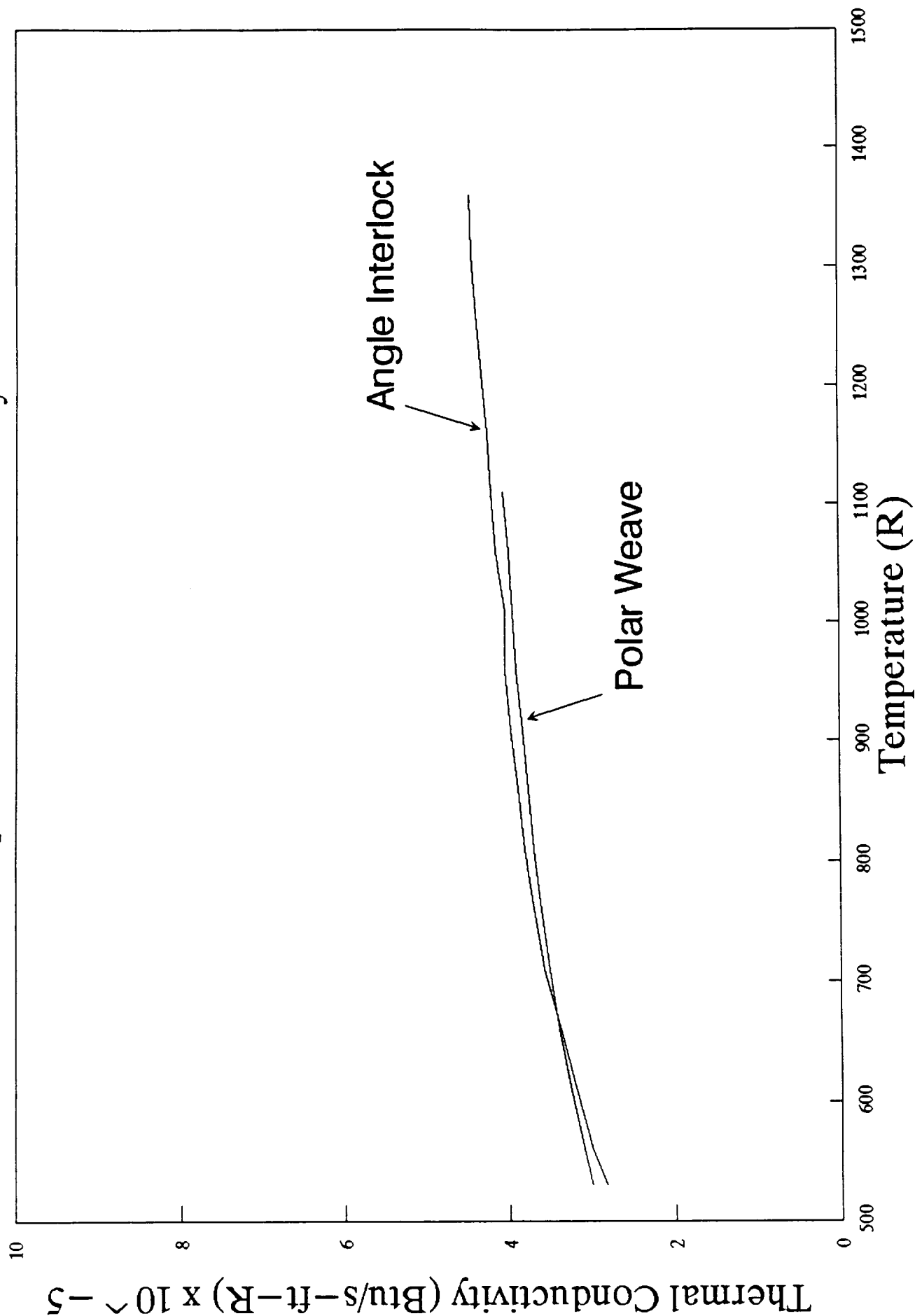


Figure 7. Thermal Conductivity of Quartz Fabric

S-glass Thermal Conductivity

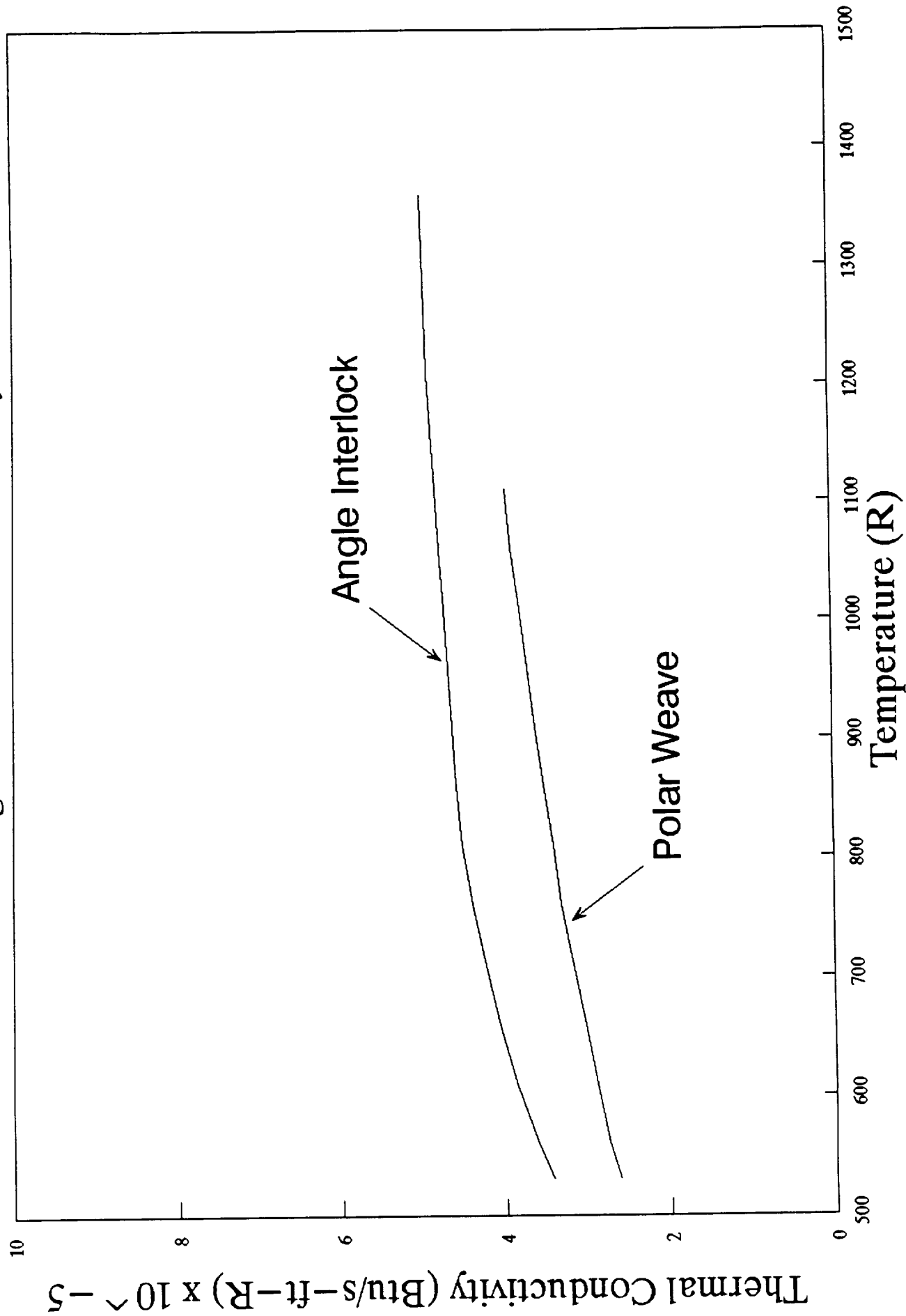


Figure 8. Thermal Conductivity of S-Glass Fabric

Kevlar Thermal Conductivity

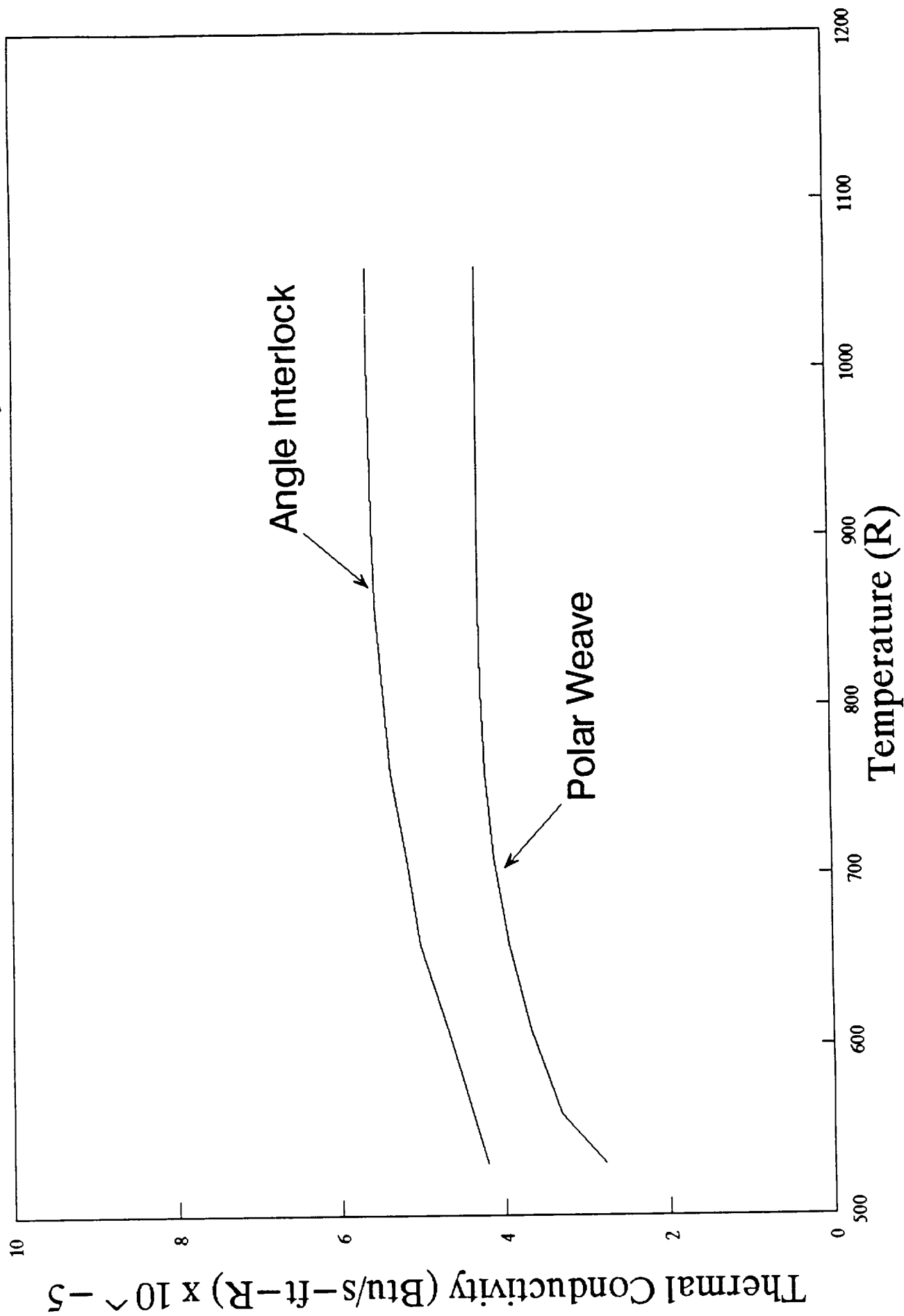


Figure 9. Thermal Conductivity of Kevlar Fabric

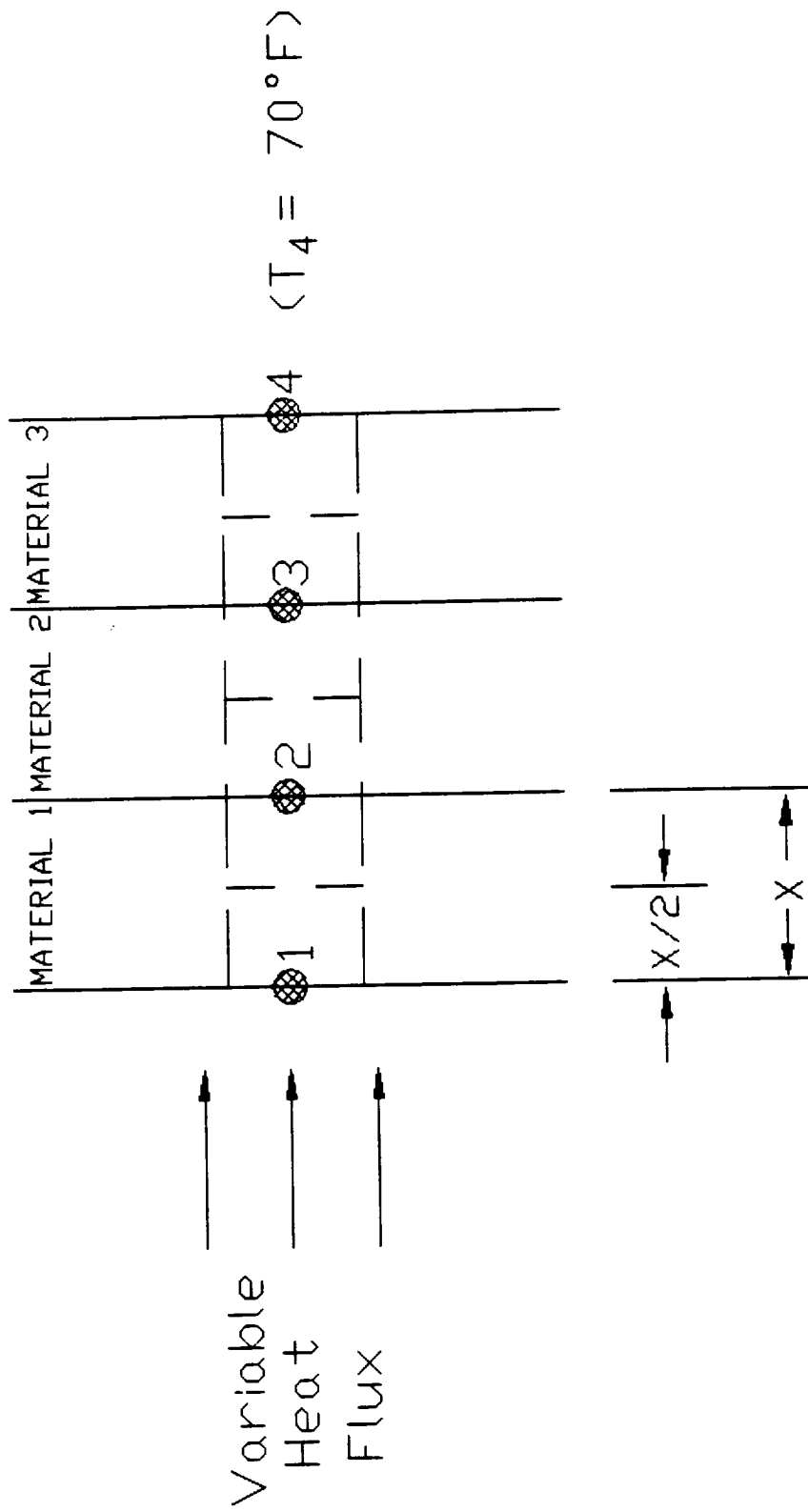


Figure 10. Four Node Configuration

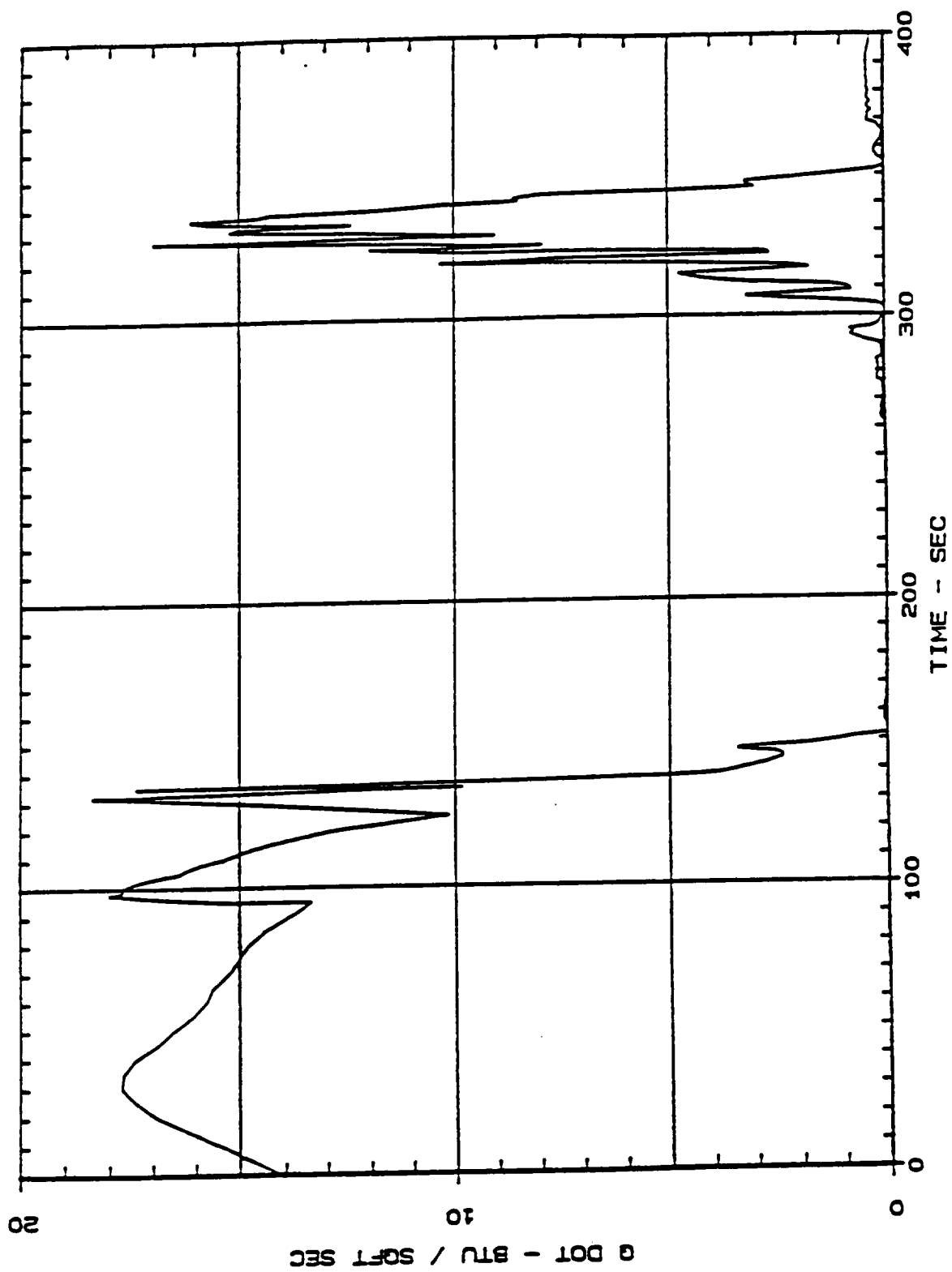


Figure 11. SRB Heat Flux Input as a Function of Time

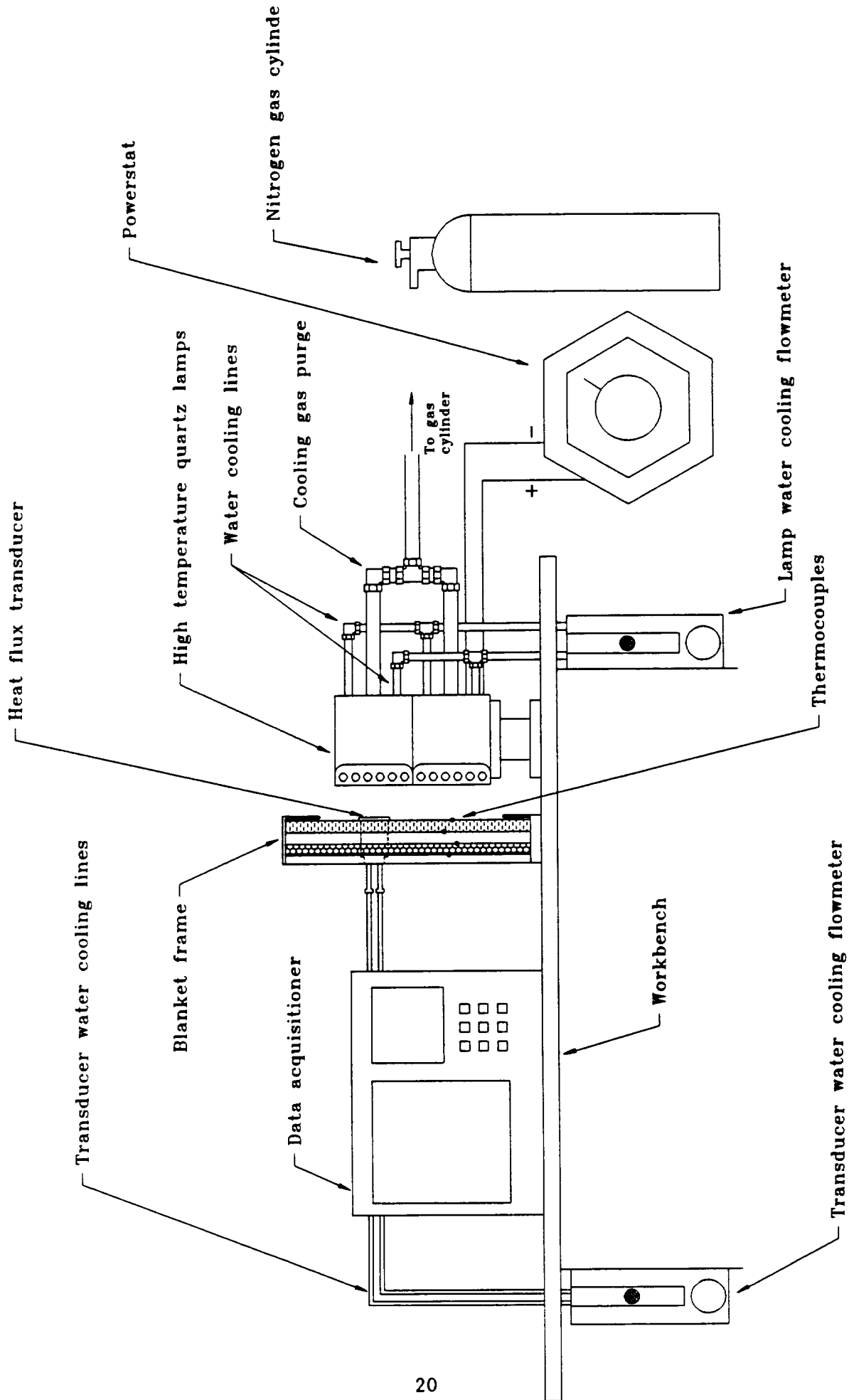


Figure 12. Schematic of Quartz Lamp Thermal Flux Facility

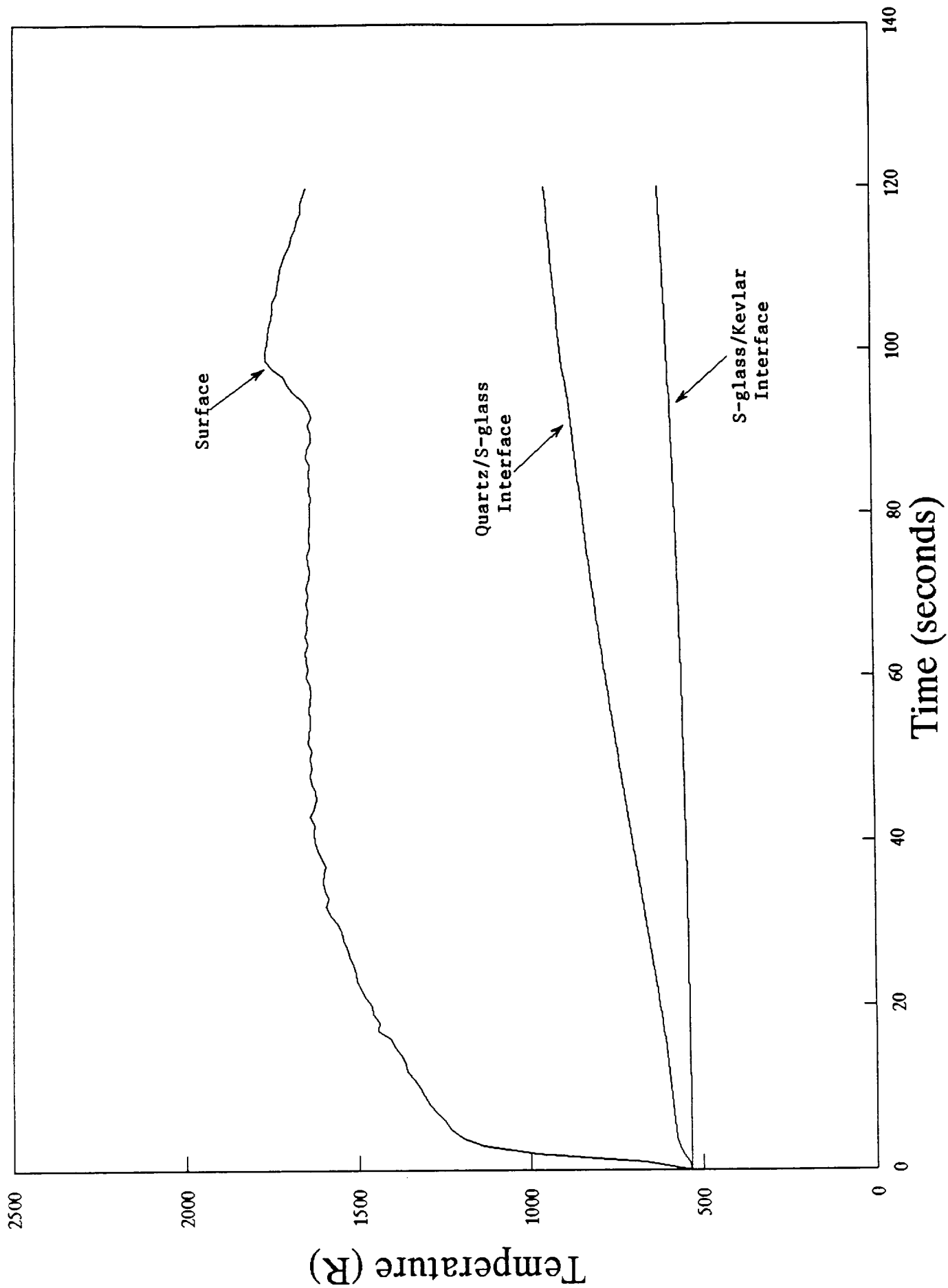


Figure 13. Quartz Lamp Test Data for 3-Piece Polar Weave Thermal Blanket

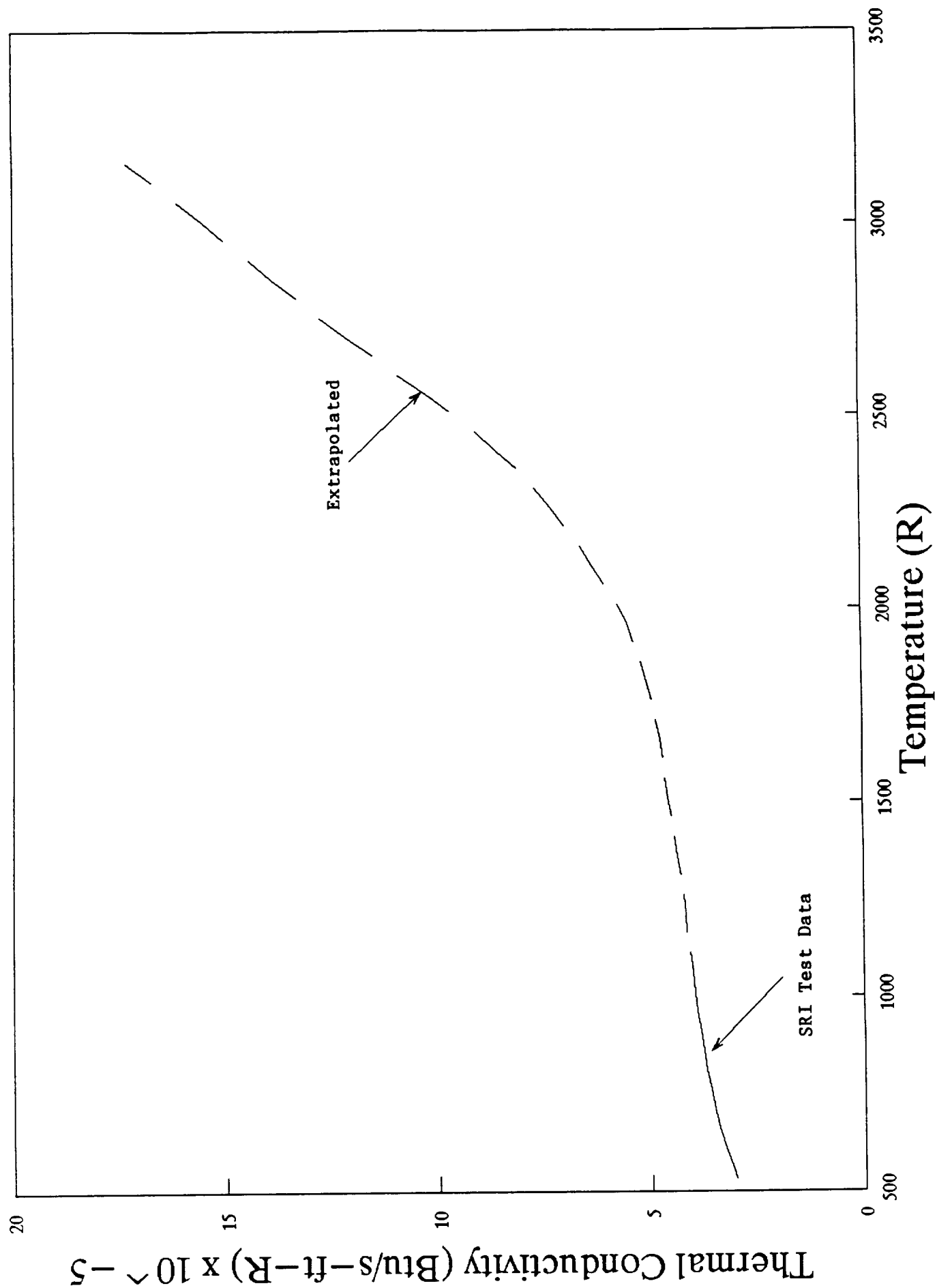


Figure 14. Extrapolated Thermal Conductivity of Polar Weave Quartz

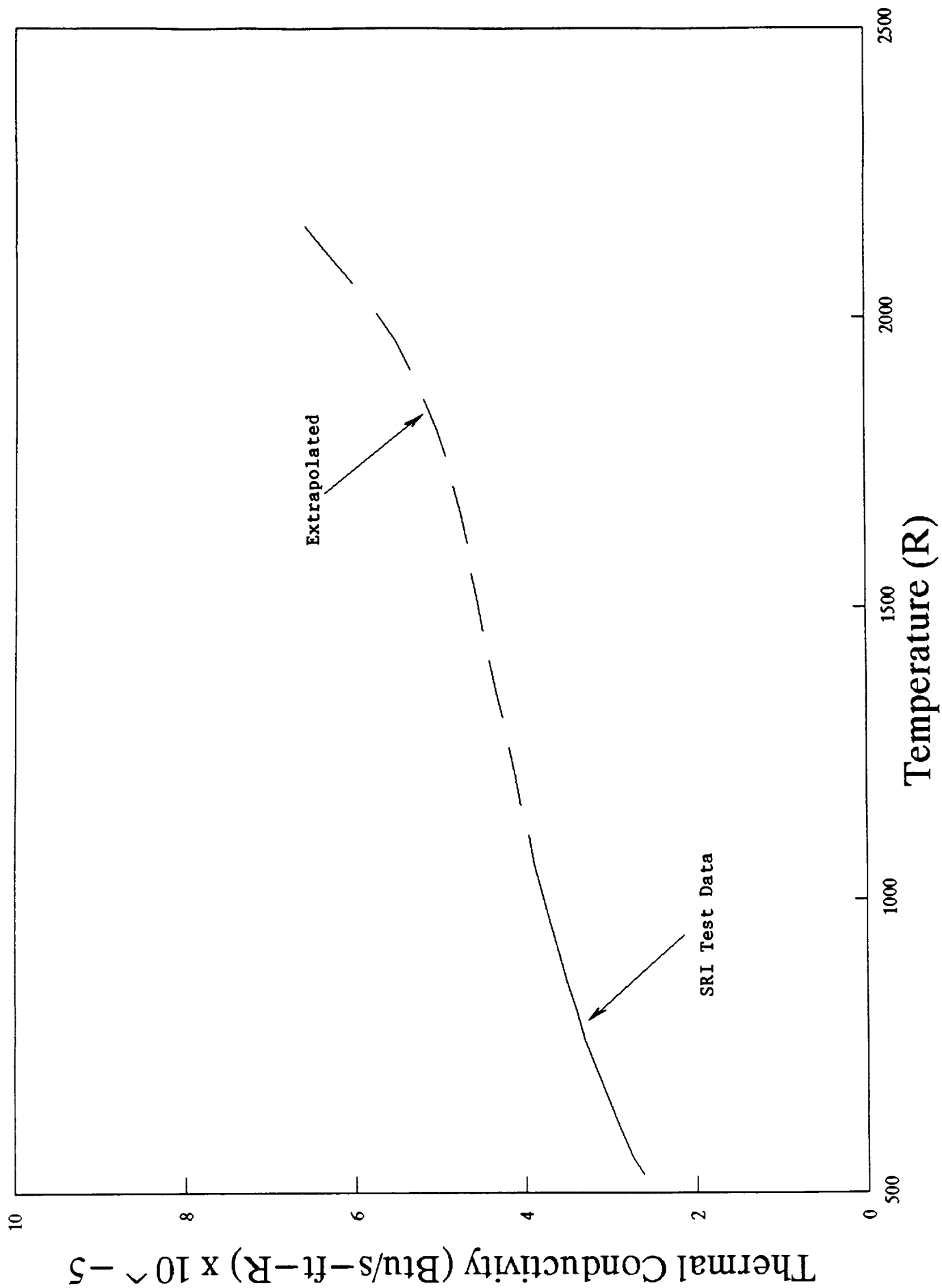


Figure 15. Extrapolated Thermal Conductivity of Polar Weave S-Glass

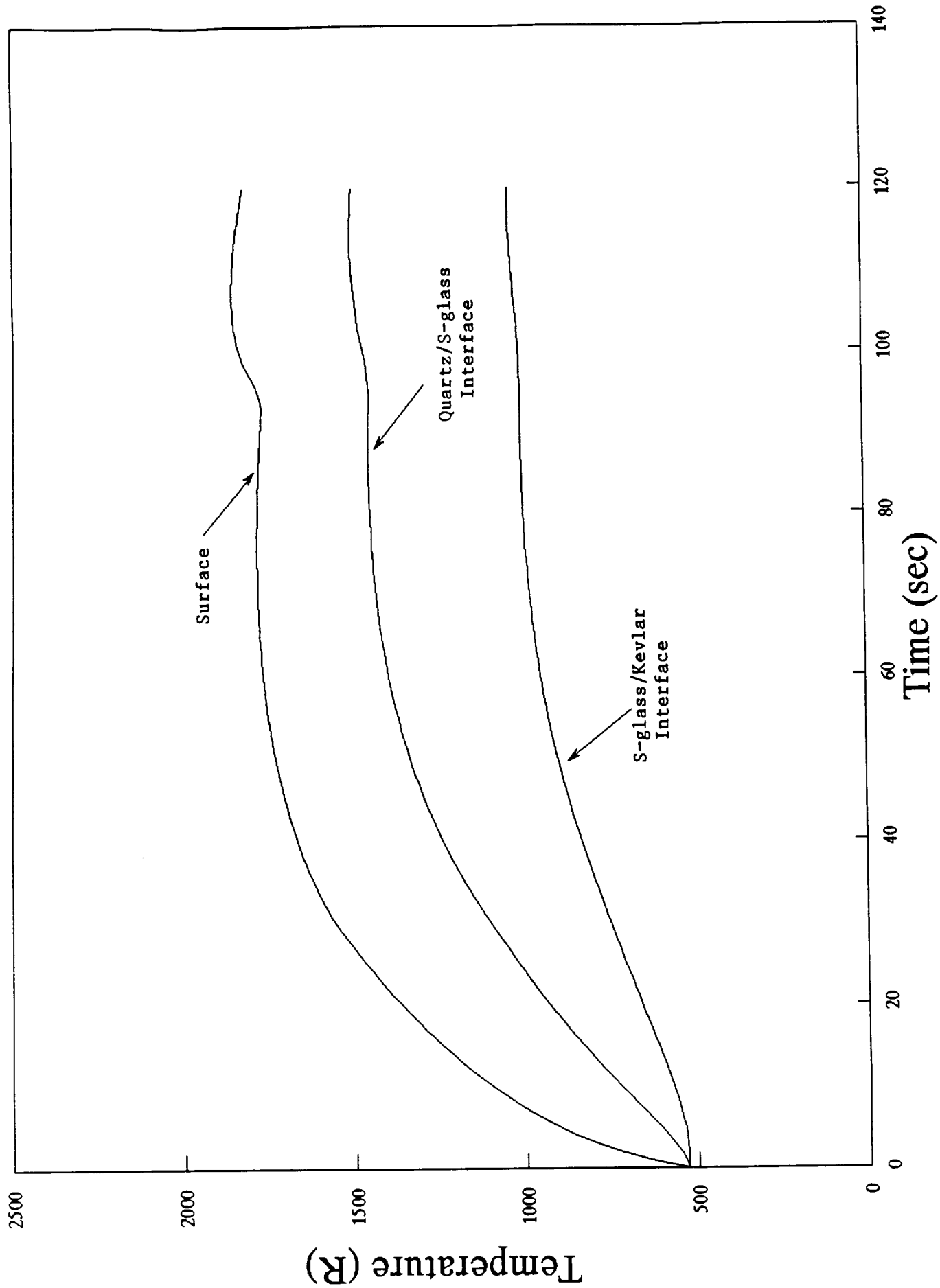


Figure 16. Thermal Analysis for 3-Piece Polar Weave Thermal Blanket

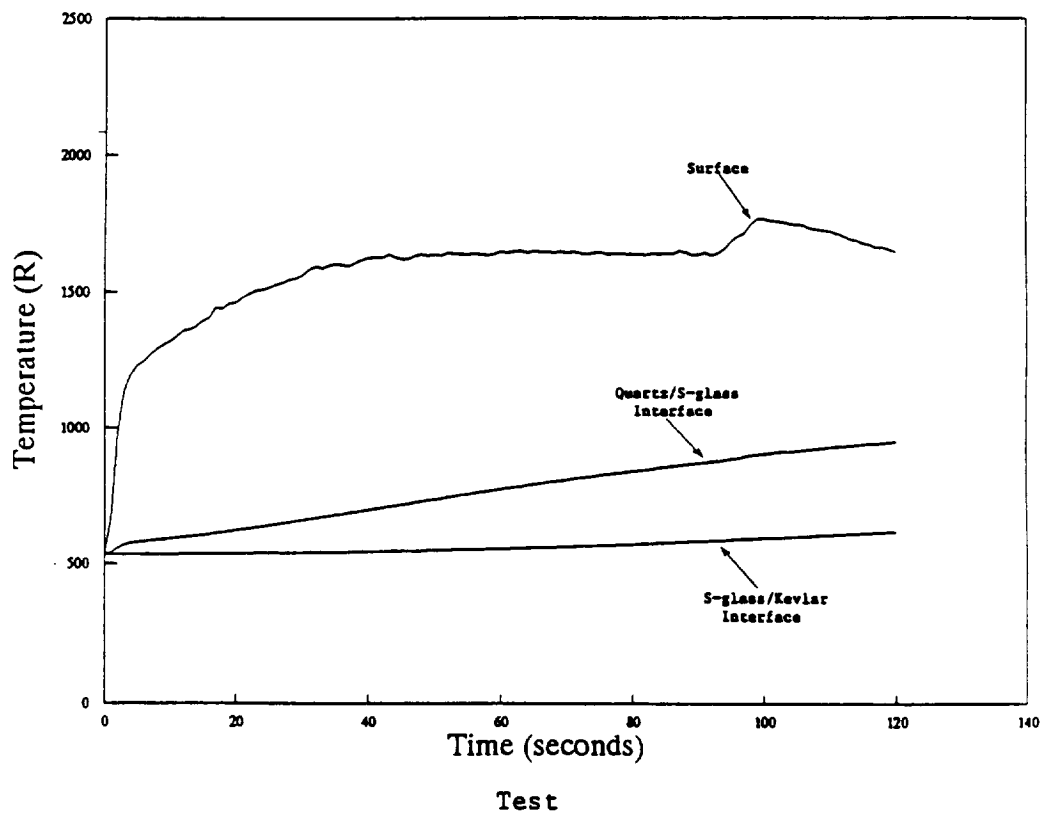
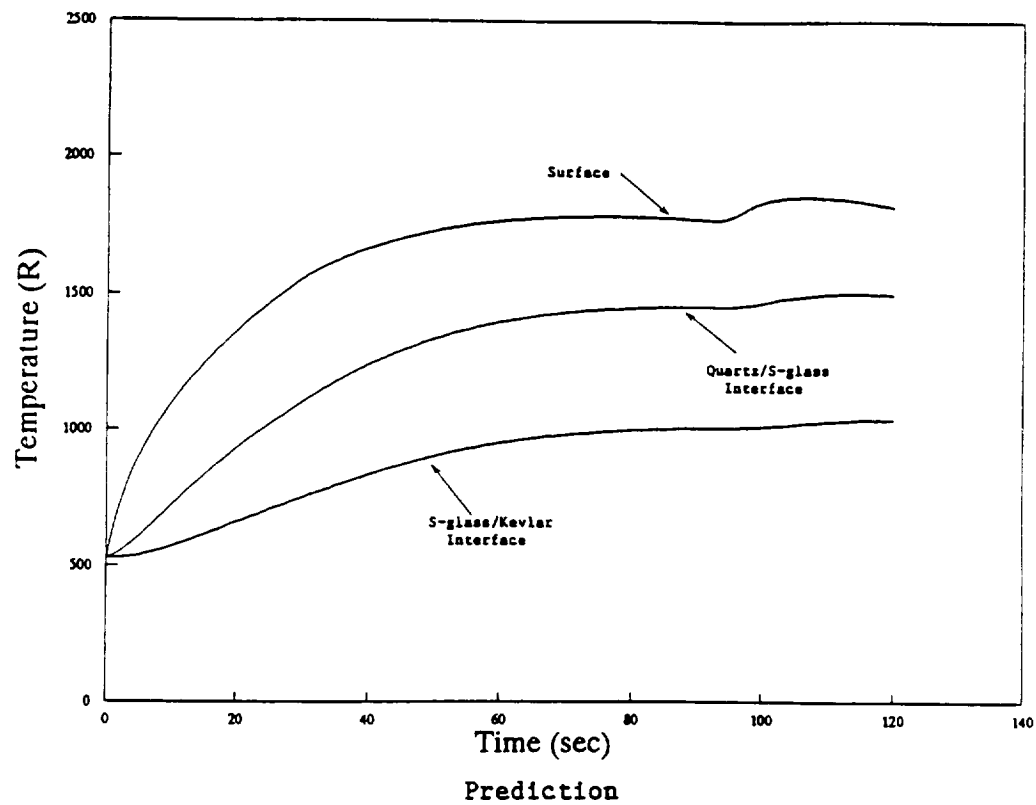


Figure 17. Comparison of Thermal Analysis to Quartz Lamp Test Data for 3-Piece Polar Weave Thermal Blanket

Table 1. Physical Characteristics of Candidate ASTC Materials

Material	Warp Count (yarn/in.)	Fill Count (yarn/in.)	Areal Weight (gm/m ²)	Areal Weight (oz.yd ²)	Bulk Density (lb/ft ³)	Construction
Angle-interlock fabric, Style ES-1742 Kevlar 29 3000 Denier, 12" wide 0.25" thick	117.0	96.0	3329.0	97.87	37.41	9 Warp x 8 Fill
Angle-interlock fabric, Style ES-2255 S-2CG 20-End Fiberglass, 12" wide 0.25" thick	117.0	96.0	6086.4	178.9	62.90	9 Warp x 8 Fill
Angle-interlock fabric, Style ES-2256 Quartz 300 2/4/4, 12" wide 0.25" thick	117.0	96.0	5458.6	161.0	53.88	9 Warp x 8 Fill
Polar Weave Fabric, Style ES-2317 Kevlar 29 3000 Denier, 12" wide, 4.5 ft. Radius 0.25" thick	117.0	96.0	3184.0	93.9	37.83	9 Warp x 8 Fill
Polar Weave Fabric, Style ES-2318 Quartz 300 2/4/4, 12" wide, 4.5 ft. Radius 0.25" thick	117.0	96.0	5221.0	154.0	54.69	9 Warp x 8 Fill
Polar Weave Fabric, Style ES-2319 20 End S-2 Glass 12" wide, 4.5 ft. Radius 0.25" thick	117.0	96.0	6802.0	200.6	61.93	9 Warp x 8 Fill

Table 2. Recommended Specific Heat of Quartz Fabric

Temperature (R)	Specific Heat (Btu/lb-R)
530	0.170
560	0.177
610	0.188
660	0.197
710	0.206
760	0.214
810	0.221
860	0.228
910	0.233
960	0.238
1010	0.243
1060	0.247
1110	0.250
1160	0.253
1210	0.256
1260	0.259
1310	0.262
1360	0.265
1410	0.267
1460	0.269
1510	0.271
1560	0.273
1610	0.274
1660	0.277
1710	0.278
1760	0.281
1810	0.283
1860	0.285
1910	0.287
1960	0.288
2010	0.289
2060	0.292
2110	0.293
2160	0.295
2210	0.296
2260	0.297
2310	0.298
2360	0.299
2410	0.300
2460	0.300
2510	0.301
2560	0.301
2610	0.302
2660	0.302
2710	0.302
2760	0.302
2810	0.302
2860	0.303
2910	0.303
2960	0.303
3010	0.304
3060	0.304
3110	0.304
3160	0.304

Table 3. Recommended Specific Heat of S-Glass Fabric

Temperature (R)	Specific Heat (Btu/lb-R)
530	0.170
560	0.177
610	0.188
660	0.197
710	0.206
760	0.214
810	0.221
860	0.228
910	0.233
960	0.238
1010	0.243
1060	0.247
1110	0.250
1160	0.253
1210	0.256
1260	0.259
1310	0.262
1360	0.265
1410	0.267
1460	0.269
1510	0.271
1560	0.273
1610	0.274
1660	0.277
1710	0.278
1760	0.281
1810	0.283
1860	0.285
1910	0.287
1960	0.288
2010	0.289
2060	0.292
2110	0.293
2160	0.295

Table 4. Recommended Specific Heat of Kevlar Fabric

Temperature (R)	Specific Heat (Btu/lb-R)
530	0.282
560	0.300
610	0.326
660	0.346
710	0.362
760	0.376
810	0.387
860	0.394
910	0.398
960	0.400
1010	0.401
1060	0.402

Table 5. Recommended Thermal Conductivity of Quartz Fabric

Temperature (R)	Polar Weave Thermal Conductivity (Btu/sec-ft-R x 10 ⁻⁵)	Angle-Interlock Thermal Conductivity (Btu/sec-ft-R x 10 ⁻⁵)
530	3.01	2.82
560	3.10	3.01
610	3.26	3.19
660	3.40	3.38
710	3.52	3.59
760	3.61	3.70
810	3.70	3.82
860	3.77	3.89
910	3.84	3.98
960	3.91	4.05
1010	3.96	4.05
1060	4.00	4.17
1110	4.07	4.21
*1160	*4.14	*4.26
1210	4.17	4.33
1260	4.21	4.40
1310	4.28	4.44
1360	4.35	4.47
1410	4.42	4.47
1460	4.49	4.49
1510	4.56	4.56
1560	4.63	4.63
1610	4.68	4.68
1660	4.75	4.75
1710	4.84	4.84
1760	4.93	4.93
1810	5.05	5.05
1860	5.19	5.19
1910	5.35	5.35
1960	5.51	5.51
2010	5.74	5.74
2060	6.02	6.02
2110	6.32	6.32
2160	6.60	6.60
2210	6.94	6.94
2260	7.29	7.29
2310	7.66	7.66
2360	8.06	8.06
2410	8.56	8.56
2460	9.05	9.05
2510	9.65	9.65
2560	10.21	10.21
2610	10.88	10.88
2660	11.46	11.46
2710	12.13	12.13
2760	12.73	12.73
2810	13.26	13.26
2860	13.87	13.87
2910	14.40	14.40
2960	14.98	14.98
3010	15.51	15.51
3060	16.11	16.11
3110	16.67	16.67
3160	17.29	17.29

*Data beyond this point extrapolated

Table 6. Recommended Thermal Conductivity of S-Glass Fabric

Temperature (R)	Polar Weave Thermal Conductivity (Btu/sec-ft-R x 10 ⁻⁵)	Angle-Interlock Thermal Conductivity (Btu/sec-ft-R x 10 ⁻⁵)
530	2.62	3.43
560	2.75	3.61
610	2.89	3.87
660	3.03	4.07
710	3.17	4.24
760	3.31	4.40
810	3.40	4.51
860	3.52	4.58
910	3.61	4.63
960	3.70	4.68
1010	3.80	4.72
1060	3.89	4.77
1110	3.96	4.81
*1160	*4.05	*4.86
1210	4.12	4.91
1260	4.19	4.93
1310	4.26	4.95
1360	4.35	4.98
1410	4.42	4.98
1460	4.49	5.02
1510	4.56	5.05
1560	4.63	5.12
1610	4.68	5.21
1660	4.75	5.30
1710	4.84	5.39
1760	4.93	5.51
1810	5.05	5.67
1860	5.19	5.83
1910	5.35	6.02
1960	5.51	6.20
2010	5.74	6.48
2060	6.02	6.74
2110	6.32	7.06
2160	6.60	7.43

*Data beyond this point extrapolated

Table 7. Recommended Thermal Conductivity of Kevlar Fabric

Temperature (R)	Polar Weave Thermal Conductivity (Btu/sec-ft-R x 10 ⁻⁵)	Angle-Interlock Thermal Conductivity (Btu/sec-ft-R x 10 ⁻⁵)
530	2.78	4.21
560	3.31	4.40
610	3.68	4.70
660	3.94	5.02
710	4.12	5.19
760	4.21	5.37
810	4.26	5.46
860	4.28	5.56
910	4.28	5.58
960	4.28	5.60
1010	4.28	5.63
1060	4.28	5.63

Table 8. Optical Properties Measurements at 530°R

Material	Wavelength Range	Zenith Angle (Degrees)	Transmittance	Average Emittance
Polar-Weave Quartz (Circumferential)	1.6-26 Microns	20(near normal)	0%	0.871
		45	0%	0.851
		75	0%	0.789
Polar Weave Quartz (Radial)	1.6-26 Microns	20(near normal)	0%	0.861
		45	0%	0.856
		75	0%	0.794

Table 9

Nodal Equations

Node 1 (given Q)	$\left[1 + \frac{2K_1 \Delta t}{\rho_1 C_1 \Delta X^2} \right] T_1^1 - \frac{2K_1 \Delta t}{\rho_1 C_1 \Delta X^2} T_2^1 - \frac{2Q \Delta t}{\rho_1 C_1 \Delta X} + T_1$
Node 2 (Composite)	$\frac{K_1}{\Delta X} \left[T_1^1 - T_2^1 \right] + \frac{K_2}{\Delta X} \left[T_3^1 - T_2^1 \right] - \left[\frac{\rho_1 C_1 + \rho_2 C_2}{2} \right] \frac{\Delta X}{\Delta t} \left[T_2^1 - T_2 \right]$
Node 3	$\frac{K_2}{\Delta X} \left[T_2^1 - T_3^1 \right] + \frac{K_3}{\Delta X} \left[T_4^1 - T_3^1 \right] - \left[\frac{\rho_2 C_2 + \rho_3 C_3}{2} \right] \frac{\Delta X}{\Delta t} \left[T_3^1 - T_3 \right]$
Node 4	Fix at some T_4

Table 10

Matrix Form of Nodal Equations

$$\begin{aligned}
\text{Node 1} \quad & \left[1 + \frac{2K_1 \Delta t}{\rho_1 C_1 \Delta X^2} \right] T_1^1 - \left[\frac{2K_1 \Delta t}{\rho_1 C_1 \Delta X^2} \right] T_2^1 - \left[\frac{2Q \Delta t}{\rho_1 C_1 \Delta X} \right] + T_1 \\
\text{Node 2} \quad & - \left[\frac{2K_1 \Delta t}{(\rho_1 C_1 + \rho_2 C_2) \Delta X^2} \right] T_1^1 + \left[\frac{2 \Delta t}{(\rho_1 C_1 + \rho_2 C_2) \Delta X} \right] \left[\frac{K_1 + K_2}{\Delta X} + \frac{(\rho_1 C_1 + \rho_2 C_2) \Delta X}{2 \Delta t} \right] T_2^1 - \left[\frac{2K_2 \Delta t}{(\rho_1 C_1 + \rho_2 C_2) \Delta X^2} \right] T_3^1 - T_2 \\
\text{Node 3} \quad & - \left[\frac{2K_2 \Delta t}{(\rho_2 C_2 + \rho_3 C_3) \Delta X^2} \right] T_2^1 + \left[\frac{2 \Delta t}{(\rho_2 C_2 + \rho_3 C_3) \Delta X} \right] \left[\frac{K_2 + K_3}{\Delta X} + \frac{(\rho_2 C_2 + \rho_3 C_3) \Delta X}{2 \Delta t} \right] T_3^1 - \left[\frac{2K_3 \Delta t}{(\rho_2 C_2 + \rho_3 C_3) \Delta X^2} \right] T_4^1 - T_3
\end{aligned}$$

Table 11

Matrix Coefficients

A_{11}	-	$[1 + 2K_1 \Delta t / \rho_1 C_1 \Delta X^2]$
A_{12}	-	$- [2K_1 \Delta t / \rho_1 C_1 \Delta X^2]$
A_{21}	-	$- [2K_1 \Delta t / (\rho_1 C_1 + \rho_2 C_2) \Delta X^2]$
A_{22}	-	$- [(2\Delta t / (\rho_1 C_1 + \rho_2 C_2) \Delta X) (K_1 + K_2 / \Delta X + (\rho_1 C_1 + \rho_2 C_2) \Delta X / 2\Delta t)]$
A_{23}	-	$- [2K_2 \Delta t / (\rho_1 C_1 + \rho_2 C_2) \Delta X^2]$
A_{32}	-	$- [2K_2 \Delta t / (\rho_2 C_2 + \rho_3 C_3) \Delta X^2]$
A_{33}	-	$- [(2\Delta t / (\rho_2 C_2 + \rho_3 C_3) \Delta X) (K_2 + K_3 / \Delta X + (\rho_2 C_2 + \rho_3 C_3) \Delta X / 2\Delta t)]$
A_{34}	-	$- [2K_3 \Delta t / (\rho_2 C_2 + \rho_3 C_3) \Delta X^2]$
A_{43}	-	$- [2K_3 \Delta t / (\rho_3 C_3 + \rho_4 C_4) \Delta X^2]$

